

Non-uniform sediment transport in rivers: numerical models and their applications

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Content

- I. Conventional numerical models of fractional sediment transport
 - Model concepts
 - Further developments
 - Various applications
- II. Artificial neural networks (ANN) for sediment transport
 - What are ANN?
 - How do ANN work?
 - Applications of ANN?
- III. Conclusion



Conventional numerical models of non-uniform sediment transport

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- Sediment transport problems in rivers are of enormous importance and require sustainable sediment management
- Numerical modeling plays a key role.





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Model concepts

- Interaction of hydromophological processes: Flow water, sediment transport and bed change (properties and elevation).
- Described by the conservation law together with several emperical formulas
- Mathematical model: coupled nonlinear equations system

Initial and boundary conditions.

- \Rightarrow Numerically solved for unsteady cases.
- Three approaches for nonuniform sediment transport



1. Multilayer



2. Size-fraction

- The bed material is divided into a number of sizefractions, each characterized by a certain diameter and by a volume percentage of occurrence in the bed material (probability).
- The sediment-transport rates depend on the bedmaterial composition, which itself depends on the history of erosion and deposition rates.



Äquivale	entdurchmesser [mm]	Benennung	Hauptfraktion
≤ 0,0002 0,0002 0,00063	- 0,00063 - 0,0020	Feinton Mittelton Grobton	Ton
0,002 0,0063 0,020	- 0,0063 - 0,020 - 0,063	Feinschluff Mittelschluff Grobschluff	Schluff
0,063 0,20 0,63	- 0,20 - 0,63 - 2,0	Feinsand Mittelsand Grobsand	Sand
2,0 6,3 20	- 6,3 - 20 - 63	Feinkies Mittelkies Grobkies	Kies (Grus)
63 200 > 630	- 200 - 630	Steine Blöcke Großblöcke	

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3. Quasi-steady condition

- Flow calculation, sediment transport and bed changes are calculated separately in one time step.
- Among others, a correct representation of the flow is a basic requirement.
 - \Rightarrow It makes sense to map the flow as precisely as possible.



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Notes on the current state of the models

- While the Navier-Stokes and continuity equations provide a well-accepted mathematical description of flow, there is no comparable model for the full interaction between flow, sediment transport, and river bed change.
- Empirical approaches with numerous parameters:
 - Bed-load: equilibrium, bed-form, Hiding-Exposure-factor,...
 - Sediment exchange: Rates of erosion and deposition, layer thickness, …
 - Constant bed porosity: (not considering infiltration, colmation, armouring...)
- \Rightarrow Sensitivity analysis, calibration and validation are needed before applying the models.
- \Rightarrow Model concept development



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Further developments

1. Non-equilibrium bed load transport



 $\frac{\partial \alpha_{bs} Q_b^*}{\partial s} + \frac{\partial \alpha_{bn} Q_b^*}{\partial n} = -\frac{1}{L_s} \left(Q_b^* - Q_e \right)$

- Conservation law for sediment exchange between bed and flow
- Actual and equilibrium bed load
- Adaptation length for bed load transport.

Computers & Geosciences 36 (2010) 792-800



Numerical modelling of non-equilibrium graded sediment transport in a curved open channel

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Considering the variation of bed-porosity and infiltration processes



$$\begin{split} &\frac{\partial\beta_{a,j}}{\partial t} = \frac{1}{E_a(1-p_a)} \left[\beta_{a,j} E_a \frac{\partial p_a}{\partial t} - \beta_{a,j} (1-p_a) \frac{\partial E_a}{\partial t} - \frac{\partial \left(\beta_{b,j} q_b\right)}{\partial x} \right] + \frac{S_{F,j}}{E_a(1-p_a)} \end{split} \quad \text{Void space of grading of the space of grading term is the space of the spac$$





Article

Advanced Numerical Modeling of Sediment Transport in Gravel-Bed Rivers

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Exemplary applications

1. Reservoir sediment management

- One method of removing deposited material from a storage area is the use of controlled flushing processes and control structures (groynes, longitudinal structures, islands, etc.)
- Numerical models can predict the hydromorphological change and optimize engineering concepts for sediment management in the reservoir.
- Various projects in Switzerland, Germany, Austria and Sudan.
- 1. Bui, M.D.; P. Rutschmann (2012): "Numerical investigation of hydro-morphological changes due to training works in the Salzach River"; Proc. of River-Flow-2012, San Jose, Costa Rica.
- 2. Bui, M.D.; P. Rutschmann (2013): **"Assessment of Sedimentation** and Flushing Efficiency for Kajbar Hydropower Project in the Nile River"; Proc. of the 35th IAHR World Congress, Chengdu, China.
- 3. Bui, M.D.; P. Rutschmann (2016): **"Numerical modelling for reservoir sediment management";** Proc. of the 6th Intern. Conf. and Exhibition on Water Resources and Hydropower Development in Asia, Vientiane, Laos.





Sediment flushing of Xiaolangdi reservoir (Source: Xinhua)

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2. Integrated model system for flood predicting

Morphological questions (e.g. the mobilization and deposition of bed load in the river course; the sediment deposition after flood events in the retention areas and flood plains) were considered:

- To evaluate the morphodynamic influence on flood simulation, and
- To provide more reliable predictions for a flood protection strategy.



Science of the Total Environment 647 (2019) 814-826



An integrated approach for investigating the correlation between floods and river morphology: A case study of the Saalach River, Germany



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3. Ecological model system

Coupling of the hydro-morphodynamic model system with the fish habitat and fish population models

GEWÄSSER

Minh Duc Bui und Peter Rutschmann

Anwendung eines numerischen Modellsystems zur Bewertung von Geschiebehaushalt und Äsche-Laichhabitaten des Hochrheins

Ein mehrdimensionales Computermodellsystem zur Simulation und Bewertung der hydromorphologischen Prozesse und der Fischhabitateignungen sowie deren Verbesserungspotenziale in großen Flüssen wird vorgestellt. Das Werkzeug zeigt die Möglichkeiten moderner Programme auf und illustriert deren Einsatz sowie Vorteile für entsprechende Untersuchungen am Beispiel des Hochrheins. Die Ergebnisse zeigen, dass mit solchen Werkzeugen geplante Flussbaumaßnahmen auf deren Erfolg und Kosteneffizienz untersucht werden können und dass Sohlen- und Habitatänderungen im Fluss nicht aus Sicht einer "Mittelwertsoptik" gesehen und beurteilt werden dürfen, sondern dass in den entsprechenden Prozessen eine hohe Dynamik stattfindet und zu berücksichtigen ist.

WASSERWIRTSCHAFT 7/8 | 2015



- The construction of the hydroelectric power stations (12) provided feedback on the hydromorphological properties.
- The High Rhine has lost most of its former typical characteristics of a mountain river (decrease in the transport capacity of the flow and bed-load).







Anzahl gefängener Äschen im Stau Ryburg-Schwörstadt 1963-2004



Decrease in grayling population (*Thymallus thymallus*)



Jahr

Fish preference and physical microhabitat characteristics (depth, flow velocity and substrate conditions) for grayling spawn in the Aare (Holzer et al., 2002)





clean gravel framework

infiltration of fine sediment

The infiltration and accumulation of fine sediments in the gravel bed affects the oxygenation of the embryos or larvae of gravel spawning fish during reproduction.

- ⇒ The proposals focus on increasing bed load transport through reservoir and improving fish habitat.
- ⇒ Models to investigate the influence of flushing and bed load addition on hydromorphodynamics and habitat suitability for grayling spawn





Mean grain diameter of the surface layer (left) and percentage of suitable areas for grayling spawn (right) before and after the 1994 flood





Variation of the weighted usable area during the spawning season



RESEARCH ARTICLE

WILEY

Development of eco-hydraulic model for assessing fish habitat and population status in freshwater ecosystems

Weiwei Yao^{1,2} I Minh Duc Bui² | Peter Rutschmann²



- Matrix-model: Age-structure Population Model
- The birth rate F_{it} and the survival rate S_{it} are functions of HSI.



Grayling-Population in the Aare





Population numbers and age structure

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Notes

- In addition to laboratory and field investigations, numerical modeling of hydromorphological processes is being used more and more for the planning and designing of technical measures in rivers.
- By forecasting developments in river morphology and the quality of the water habitat, a numerical model system offers the possibility
 - to assess the impact of sediment transport on planned hydraulic engineering and water management projects and their consequences,
 - to optimize measures, and thus to minimize technical, economic and ecological problems in operation.
- Due to the complexity and dynamics of hydromorphological processes,
 - there is still no uniform description of the processes and
 - the model concepts for the description of the fractionated sediment transport have a strong empirical character:
 - ✓ Formulas with parameters have been developed in laboratory/field tests under specific conditions.
 - ✓ Prior knowledge of the nature of the relationships between the data is necessary.
 - \Rightarrow Limited areas of application



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Artificial Neural Networks for sediment transport

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• Artificial Neural Networks (ANN):

Well-known applications are text, image and speech recognition, where computers examine the data for certain characteristics in order to make an assignment.

- Games with complex decisions, vehicle controls, etc.
- Can approximate any relationships to create simulations and determine forecasts, e.g. in water management, weather forecasting, medical diagnostics or on the stock markets.



\Rightarrow Develop ANN models for sediment transport problems

What are ANN?



 Mathematical replicas of biological nervous systems: the ability to respond (response) to signals from the environment (stimulus)

 $\begin{array}{rl} \mbox{Receive inputs} & \Rightarrow \\ \mbox{Process data (through learning process)} & \Rightarrow \\ \mbox{Provide outputs.} \end{array}$





- ANN: made up of a large number of artificial neurons and an associated network structure.
 - Neurons are defined as mathematical functions that filter the signal through the network.
 - In an ANN, neurons are arranged in layers.
 Each neuron is connected to other neurons via weighted connections



The choice of network topology is of particular importance and depends on the problem.

Functioning of a neuron



- Biological Neuron:
- Signals are received via the dendrites and forwarded to the cell body and added up.
- If the signals have exceeded a certain threshold value, the cell nucleus is activated, the signals are analyzed, evaluated and finally forwarded via the axon and then transmitted through the synapses to the dendrites of the next neurons.
- The biological learning process takes place through the adaptation of the connections (synapses) between the nerve cells.





- Artificial Neuron:
- A neuron receives its weighted inputs, which are combined and passed through a transfer or activation function to produce the output.
- Common activation functions are the logistic (*logsig*) and the hyperbolic tangent (*tansig*).
- The weights and thresholds are adjusted and defined by the learning or training process.

Learning/training process



- An ANN learns for a given input by modifying itself according to a fixed rule (the learning rule) to produce a desired output.
- Principally possible learning processes:
 - ✓ Develop new connections and delete existing one,
 - ✓ Change connection weights and thresholds,
 - ✓ Aktivierungsfunktionen abwandeln,
 - ✓ Develop new neurons or delete existing neurons (neurons, layers and their connections).
- Network training finds a set of weights and thresholds that minimizes error for all data in the training set:

$$E(W) = \frac{1}{2} \sum_{i=1}^{L} (d_i - y_i)^2 = \frac{1}{2} \sum_{i=1}^{L} \varepsilon_i^2$$
$$W^{(k+1)} = W^{(k)} + \Delta W^{(k)}$$



- Model performance depends on:
 - ✓ Initialization of weights and thresholds
 - ✓ Network architecture: inputs, number of layers, number of hidden neurons, activation functions
 - \checkmark Training method etc.

Levenberg-Marquardt-Algorithm:

$$\Delta W = -\left[J^T J + \mu I\right]^{-1} J^T \varepsilon$$

- J Jacobi-Matrix,
- I Identity matrix,
- μ Marquardt-Parameter

ANN as Predictor





Journal Journal of Applied Water Engineering and Research > Volume 3, 2015 - Issue 2

Original Articles

Contraction scour estimation using datadriven methods

Minh Duc Bui 🔄, Keivan Kaveh, Petr Penz & Peter Rutschmann



 $\overline{d_s} = \frac{d_s}{b_1}; \overline{d} = \frac{d_m}{b_1}; \overline{h} = \frac{h_1}{b_1}; \overline{b} = \frac{b_2}{b_1}; \overline{Fr} = \frac{v_1}{\sqrt{\Delta g d_m}}; \Delta g = \left(\frac{\rho_s - \rho}{\rho}\right)g$ $\overline{d_s} = F\left(\overline{d}, \overline{Fr}, \overline{h}, \overline{b}, \sigma_s\right)$



- 182 measured data sets for the equilibrium depth of contraction scour
- Inputs and Output
- Network topology: Feedforward Multilayer Perceptron (MLP)
- Training und Test: Trial-and-Error







Results statistically analyse

	ANN	Dey & Raikar	Richardson & Davis	Komura	Lim & Cheng
R	0.965	0.836	0.853	0.790	0.748
RMSE [m]	0.015	0.035	0.054	0.170	0.048
MAE [m]	0.006	0.013	0.028	0.084	0.016

- The importance of the individual input parameters was checked with a sensitivity analysis.
- ⇒ The contraction ratio is the most sensitive parameter, followed by the impact of the formation of the pavement layer for non-uniform sediments.



$$(\overline{d}_{s})_{pr} = LW^{21} \times \operatorname{tansig} \left[IW^{11} \times \begin{bmatrix} \overline{d} \\ \overline{Fr} \\ \overline{h} \\ \overline{b} \\ \sigma_{g} \end{bmatrix}_{pr} + b^{1} + b^{2} \right]$$

- The trained network maps non-linear inputoutput relationships in complex systems.
- In the form of a matrix equation

IW ^{1,1} =	-61.7540 -36.1590 12.5222 112.9620 -26.4800 128.5200	0.7590 - 1.9702 - 0.0331 1.4127 0.0029 - 1.6282 1.1852	16.0804 -18.4141 -14.5885 2.7952 13.6386 8.8843 12.4488	-2.2736 3.2689 8.0144 -5.7359 8.1508 -0.1999	-1.3754 3.0961 -0.2061 -1.0236 -0.0300 0.2204 0.6407	,
	65.1840	-1.1853	-12.4488	-0.9877	-0.6407	

$$LW^{2,1} = \begin{bmatrix} 0.0276 & -0.0258 & -0.0726 & 0.0241 \\ & -0.0312 & 0.2775 & -0.0295 \end{bmatrix},$$

$$\vec{b}^{1} = \begin{bmatrix} 2.1017 \\ 1.7659 \\ -0.0878 \\ 0.1017 \\ -7.2220 \\ 4.7620 \\ 3.9530 \end{bmatrix},$$
$$b^{2} = [-0.1876].$$



Physical suitability of the model

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Application of artificial neural networks for river regime M.D. Bui¹, D. Huber¹, K. Kaveh¹, A.M. da Silva², P. Rutschmann¹

$$B = LW^{B} \times \operatorname{tansig}\left[IW^{B} \times \begin{bmatrix} Q_{1} \\ Q_{2} \\ D_{1} \\ D_{2} \end{bmatrix} + \vec{b}^{B} + c^{B} \qquad S = LW^{S} \times \operatorname{logsig}\left[IW^{S} \times \begin{bmatrix} Q_{1} \\ Q_{2} \\ D_{1} \\ D_{2} \\ B \end{bmatrix} + \vec{b}^{S} + c^{S} \right] + c^{S}$$

$$h = LW^{h} \times \operatorname{tansig}\left[IW^{h} \times \begin{bmatrix} Q_{1} \\ Q_{2} \\ D_{1} \\ D_{2} \\ B \end{bmatrix} + \vec{b}^{h} \right] + c^{h}$$

$$\frac{RMSE}{17.7} \qquad 0.3$$

$$R \qquad 0.990 \qquad 0.970$$

$$\frac{SRMSE}{1.7.7} \qquad 0.3$$

$$\frac{SRMSE}{1.7$$

 Based on 509 observed datasets, an MLP model for predicting flow regime characteristics was developed.

 Die MLP-Leistung wurde mit der thermodynamischen Entropietheorie (Y&S) und der Stabilitätstheorie (J&W) verglichen.

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S[-] 0.0019

0.929

0.0053

0.866

0.0050

0.842





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Article Variability and Trend Detection in the Sediment Load of the Upper Indus River

Sardar Ateeq-Ur-Rehman * 🗅, Minh Duc Bui 🗅 and Peter Rutschmann 💿

- Estimation of the sediment load for boundary conditions in numerical models.
- Very difficult to estimate accurately in an area with a strong hysteresis phenomenon and a disproportionate spatio-temporal trend between water runoff and suspended sediment rate.
- Development of an ANN model combined with Discrete Wavelet Transform (WAANN).



$$f_{i} = A_{M,i} + \sum_{m=1}^{M} \sum_{n=0}^{(2^{M-m}-1)} W_{m,n} 2^{\frac{m}{2}} \Psi \left(2^{-m}i - n\right)$$
$$f_{i} = A_{M,i} + \sum_{m=1}^{M} D_{m,i}$$

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- WA-ANN gives good results for the extraordinary events and fills the gap between the intermittent measurements of the suspended sediment concentration (SSC).
- Analysis of the temporal change in sediment transport rates (SSL) and water runoff using non-parametric trend tests.

International Journal of Sediment Research 32 (2017) 340-350



Original Research

A comparative study of three different learning algorithms applied to ANFIS for predicting daily suspended sediment concentration

Keivan Kaveh*, Minh Duc Bui, Peter Rutschmann



Long short-term memory for predicting daily suspended sediment concentration

Keivan Kaveh¹ · Hamid Kaveh² · Minh Duc Bui¹ · Peter Rutschmann¹



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IOSR JOURNAL OF MECHANICAL AND CIVIL ENGINEERING (IOSR-JMCE)

e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 14, Issue 3 Ver. V. (May - June 2017), PP 18-32 www.iosrjournals.org

Performance Analysis Of Different Model Architectures Utilized In An Adaptive Neuro Fuzzy Inference System For Contraction Scour Prediction

Minh Duc Bui¹, Keivan Kaveh¹, Peter Rutschmann¹ ¹(Institute of Hydraulic and Water Resources Engineering, Technical University of Munich, Germany)

Testing different ANFIS networks and training methods.

⇒ Using the zero-order Takagi-Sugeno model with 4 bell-shaped membership functions for each input, the Levenberg-Marquardt algorithm for training yields best results for contraction scour depth.





New concept for hydromorphological modelsystems



$$\frac{\partial z_{\flat}}{\partial t} = -\frac{1}{1-p} \frac{\partial q_{\flat}}{\partial x}$$

az, c	(-) ² z, 0	C(z) = z	1	∂q₅
at +C	$\frac{(z_b)}{\partial x} = 0$	0(25)-	1 – p	∂z₅

- Using Empirical Formulas to Calculate
 Sediment Transport Rates in Exner's Equation :
 - The results of the different formulas often vary widely.
 - In many cases unsatisfactory morphological changes are predicted.





E-proceedings of the 36th IAHR World Congress 28 June – 3 July, 2015, The Hague, the Netherlands

Integrating artificial neural networks into hydromorphological model for fluvial channels

MINH DUC BUI (1), KEIVAN KAVEH (2) & PETER RUTSCHMANN (3)



 $U_{(i-1)}^{n}$

 \rightarrow Stable numerical solution with a large time st (3000 times larger than using conventional numerical methods)





Advances in Engineering Software 132 (2019) 18-28



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journal homepage: www.elsevier.com/locate/advengsoft





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Notes

- Using appropriate network architectures and training processes, the ANN models can be developed for different sediment problems.
- The estimates of the ANN models are significantly better than those of traditional approaches with lower error (RMSE) and higher correlation coefficient (R).
- The equations obtained from the ANN can also be easily applied to estimate the properties of sediment transport under other hydromorphological conditions.
- A coupling of ANN methods with conventional hydromorphological models provides promising results.
- ANN can become a useful tool for sediment transport calculation.



III.

Conclusion

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- Sediment transport problems in rivers are of enormous importance and require sustainable sediment management.
- Numerical modeling plays an important role, but the sediment transport models have a strong empirical character.
- The full interaction between flow, sediment transport and river bed modification remains a challenging research topic.

• For the future, the following challenges arise for the modelers:

- 1. The numerical models have to be completed in terms of processes. Both new experimental approaches and advanced numerical models are required.
- 2. The numerical models can be used in larger areas over longer periods of time. This requires
 - the coupling of hydromorphodynamic models with hydrological models and
 - the use of high-performance computers in combination with specially adapted software.



- **To 1.**: **Refinement of the sediment transport approaches** can be improved by using highresolution numerical simulations and ANN models.
 - Application of CFD-DEM-Modellsystem:
 - the phase motion of discrete particles is determined by the Discrete Element Method (DEM) using Newton's laws of motion.
 - ✓ Water flow can be solved using traditional Computational Fluid Dynamics (CFD).
 - The interactions between the flow and the sediment transport can be modeled according to Newton's third law.
 - The use of an alternative, numerical method capable of modeling both the flow and the sediment phase with full complexity and process accuracy:
 - ✓ z.B. Smoothed Particle Hydrodynamics (SPH), Multiphase Particle in Cell (MPPIC), or Cellular Automation (CA) approach.
 - These methods allow both phases to be modeled with the same approach, correctly simulating both the flow including turbulence and a fractionated bed load phase with all collision effects while preserving mass, momentum and energy.



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Article

Combination of Discrete Element Method and Artificial Neural Network for Predicting Porosity of Gravel-Bed River

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Article

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water

The Prediction of Fine Sediment Distribution in Gravel-Bed Rivers Using a Combination of DEM and FNN

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check for updates

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To 2.: Large-scale numerical models

- A hydrological-hydromorphodynamic coupling with a high process resolution on the highperformance computers represents the basic prerequisites for recording the runoff events and the morphological development more realistically and for approaching future planning of flood protection measures.
- To carry out simulations efficiently and in a reasonable time, it is necessary to optimize the numerical methods of the computer code and to increase the computing power of computers accordingly.
 - ✓ To be able to use multi-core systems, it is necessary to parallelize the calculations on different levels. In other words, you divide a task into smaller subtasks, which are then handled by different processor cores.
 - Large study areas can be broken down into different sub-areas, in which all variables are calculated in parallel on the individual computers in each calculation step. At the end of the calculation step, the results are merged (known as the domain decomposition method).
 - Modern graphics cards have a large number of cores in the graphics processing unit (GPU, Graphics Processing Unit), which can perform calculations completely independently of one another. In addition, GPUs are designed for massive parallelization, so they can provide enormous computing power.



Thank you for your attention!